Texas Abacus

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"What a Wonderful World!" Colorful Cultural Activity for the Whole Family (1) – Build a Texas Abacus and Watch Your Kids Fall in Love with Arithmetic

Chapter 1: Introduction

Remember, the purpose of any project in this series is for the whole family to have fun. Therefore, if you come across any difficulty at any time, do not get frustrated. Instead, take to the Internet for help.

Before anything, let me show you the finished product first. (See Figure 1.)

Abacus originated from China. Traditionally, all the beads are of the same shape, size, and color and all columns contain the same number of beads. But if you look closely in the picture, you will see that each column has its own unique color. There are, also, three columns that contain only one single bead. I will tell you what the advantages are:

1) The columns are color-coded, meaning, you can designate and distinguish the place-values more easily. You can also work on decimal numbers, which are numbers containing the decimal point.
2) Each of the three single-bead columns is there to designate either the number is positive or negative. Traditionally, there is no way to designate negative numbers in an abacus.
3) You can also see there are three parts. They are actually three abacuses side by side. I do this so that the two numbers to be operated on can also be represented, with the operations usually done and the result shown on the last part. The last part is the largest of the three areas to the right.

Figure 1: a Texas abacus.

With these improvements and advantages in mind, therefore, I gave it the name 'Texas Abacus.'

Please take some time to get familiar with the abacus in the picture before you continue to the next part of the project.

The Box

To do the project, you will first need a box. If you are not sure you can do it, please ask your husband to make it for you. It is actually quite simple[1].

Length: 12"

Width: 5-½"

[1] Remember: we are not building a precision tool; therefore, a little deviation from the measurements should not be that big a deal. Or, you can have your own design – such as how to cut and join the sides, however is convenient for you -- as long as the finished product looks similar and serves the same purposes.

Figure 2: the box.

Height excluding the base board: 1-½”.

The four walls of the box and the divider wall inside are of the same thickness: ½”

From the top edge to the center of the diver wall: 2”

From the center of the diver wall to the bottom edge: 3-½”

Before you construct the box, drill the holes for the strings first (top wall, divider wall, and bottom wall). There will be 26 strings with a distance of 1cm between each neighboring pair. (Refer to the first picture). The holes will be drilled horizontally 3/16” from the top of the wall. The 5/64” drill bit will be the most ideal.

Once you have drilled the holes and constructed the box, give it a protective coating of your favorite color. For me, of course, I chose the pecan color.

Stringing the Beads

The stringing is easy. I used 30-lb fishing line. You can do whatever way you want. I did two neighboring strings at the same time but did it twice. Therefore, each column of beads was doubly stringed to add to the strength.

For the beads, I used the beads purchased from Walmart that had been sitting in my house for quite some time. You can choose your own colors; for example, which color you use for place value 1’s column, which color for place value 10’s column, etc. Because each step along the way, the user is actually pitting one digit in a number against one digit in another number, color-coding will greatly facilitate your identifying the correct digits you are working on and also to locate the correct places where you put the answers on. Yes, along the way, each step has its own answer. It is the development of these work-in-process answers along the way that will eventually materialize into the final answer the question is asking for.

I particularly want to point out that in the abacus I made the colors are repeating themselves in exactly the same order, because the same colors represent the same place values. The only exception is that in the largest area, because there are five more digits than the other two areas, these five extra digits are represented by their own unique colors or shapes or sizes.

Once you understand the whole idea, you may proceed to string the beads. I am sure you will have a lot of fun. But be patient. It may take up to three or four hours. Tie the ends tightly with three simple knots and apply a tiny bit of glue before cutting the extra lengths off.

Remember, this beautiful instrument will not only let your kids visualize numbers, it will actually let them touch the numbers. What a wonderful way to learn arithmetic!

Chapter 2: How Can Five Beads Represent 0 through 9?

From reading my previous writing, you might have already noticed that each column of the abacus has five beads. So the question is: how can five beads represent numbers 0, 1 etc. through 9?

To answer the question all you need is pay attention to the divider wall or bar, with the columns above the bar having one bead only each, and the columns below the bar having four beads each. (The only exceptions are the three columns each of which contains only one single bead. As I explained earlier, these three single red beads are there to designate negative numbers and you can ignore them for now.) When Chinese invented the abacus, and this was the ingenious part, they designated the value of the bead above the bar as a 5. With that special bead, they only needed five beads to represent any values 0 through 9. Because:

0 = 0 (no bead is moved next to the bar)

1 = 1 (with one bead below the bar moved next to the bar)

2 = 1 + 1 (with two beads below the bar moved next to the bar)

3 = 1 + 1 + 1 (with three beads below the bar moved next to the bar)

4 = 1 + 1 + 1 + 1 (with four beads below the bar move next to the bar)

5 = 5 (with the bead above the bar moved next to the bar with all other beads pushed back away from the bar)

6 = 5 + 1 (with the bead above the bar moved next to the bar and one bead below the bar moved next to the bar)

7 = 5 + 2 (with the bead above the bar moved next to the bar and two beads below the bar moved next to the bar)

8 = 5 + 3 (with the bead above the bar moved next to the bar and three beads below the bar moved next to the bar)

9 = 5 + 4 (with the bead above the bar moved next to the bar and four beads below the bar moved next to the bar)

How Is the Value 10 Represented?

When we add one more, because there is no more bead left in the first digit column, which has the place value of 1, because all the beads there are used, then we have a carry of one, meaning, we add one to the next digit higher up and push all the beads in the column of the first digit back away from the bar.

Do the operations begin to make sense now? All the terms a math teacher wants the students to understand and get familiar with come into play now: place value, carry, digit, first digit, digit higher up, etc. They become no longer abstract or empty terms. With an abacus, the students understand them immediately, because the students get hands-on experiences with them while their fingers are interacting with the moving abacus beads.

The following pictures show you all the numbers from 0 all the way to 10.

Figure 3: representing 0 -- no bead is moved next to the bar

Figure 4: representing 1 -- one bead below the bar

Figure 5: representing 2 -- two beads below the bar

Figure 6: representing 3 -- three beads below the bar

Figure 7: representing 4 -- four beads below the bar

Figure 8: representing 5 -- one bead above the bar

Figure 9: representing 6 -- one bead above and one bead below the bar

Figure 10: representing 7 -- one bead above and two beads below the bar

Figure 11 representing 8 -- one bead above and three beads below the bar

Figure 12: representing 9 -- one bead above and four beads below the bar

Figure 13: representing 10 -- one bead below the bar in the next higher or the 10’s digit.

In the next chapter, we will do more fun stuff with our Texas Style Abacus. But before I let you go, let’s do one thing more:

Use your calculator (Yes, an abacus is a calculator!) to represent the birthday of the United States of America: (Wait! Do not look at the answer first. Work on you own and then compare your answer with mine.)

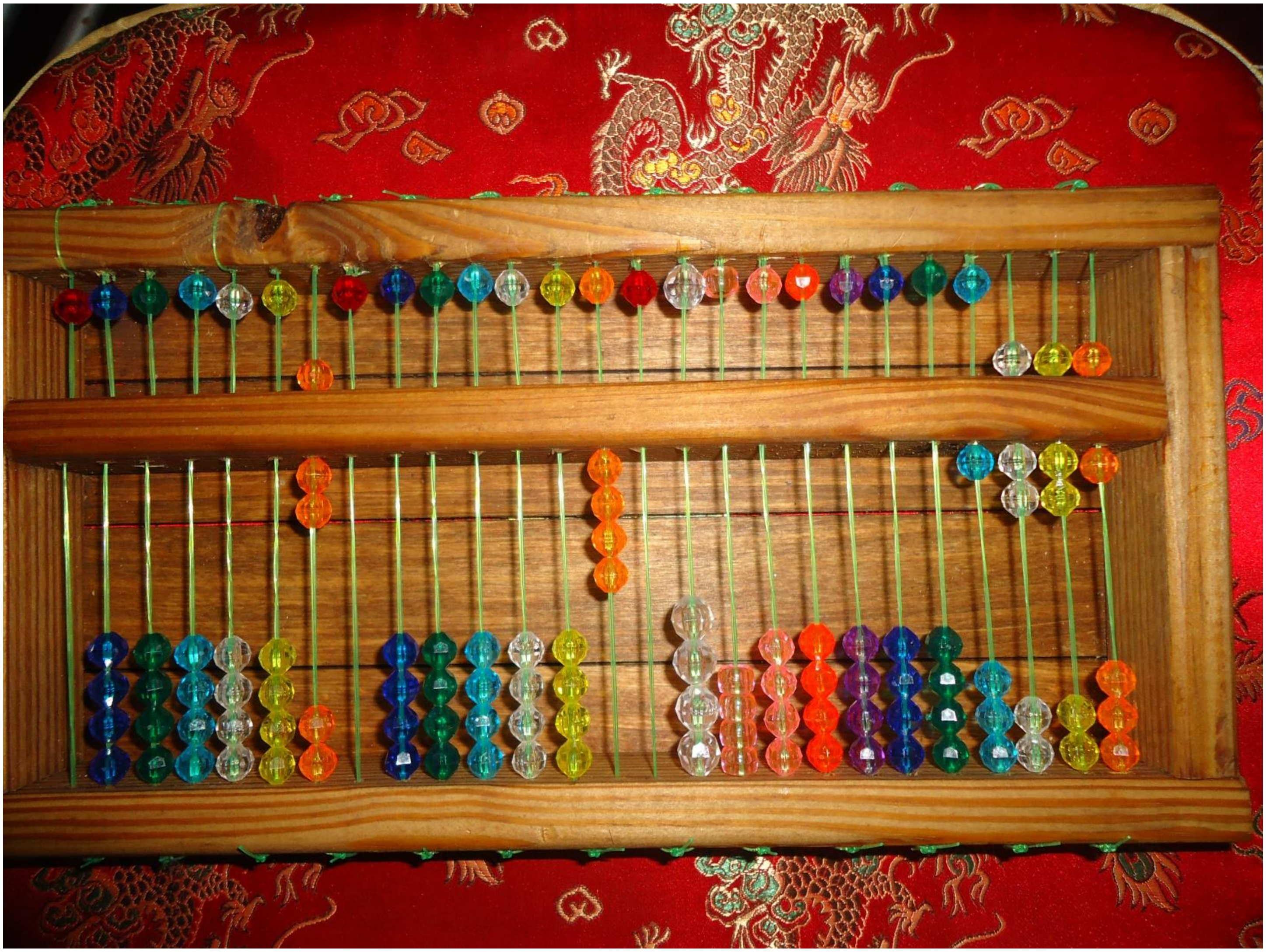


Figure 14: see how proudly the birthday of the United States of America – 7-4-1776 – is displayed on this Texas abacus!

Until next time, have fun! Bye!

Chapter 3: Mental Exercise and Fun

The abacus has been used in China for accounting purposes for at least one thousand years. Even today, some people still claim that the abacus can do faster than the electronic calculators. But for the rest of us – the common people – the claim that the abacus can do faster than the electronic calculators is really not important.

For the rest of us, operating the abacus is a good mental exercise. More importantly, it will guarantee that the user understands the basic arithmetic operations inside out. I am 100% sure that all the math teachers in the world will welcome this news. Nowadays, the students rely on the electronic calculators so much that they barely understand the mechanism of the basic arithmetic operations. The problem will get seriously worse when students are expected to learn more advanced operations. Then they immediately hit the ceilings because in the past they could get the results they wanted without even knowing why and how – except hitting the buttons. Facing more advanced operations, they are immediately lost, because the meanings of the operations have become too vague for them to grasp.

Some abacus experts have certain shortcuts that will enable them to get the results very quickly. But those skills are not my focus. **What I know will be the most beneficial for the user is that while using the abacus to do arithmetic, the user is actually doing the *longhand* arithmetic operations – but with a big difference: instead of pencil and paper, the user *spells out the details by repositioning the beads*.**[2]

Can you imagine anything so clean and green?

Two Tables, Two Sets of Rules, and One Warning

To cover the basic arithmetic operations on two numbers of various lengths, with or without the decimal point, positive or negative, i.e. addition, subtraction, multiplication, and division involving any two digital numbers, the user needs:

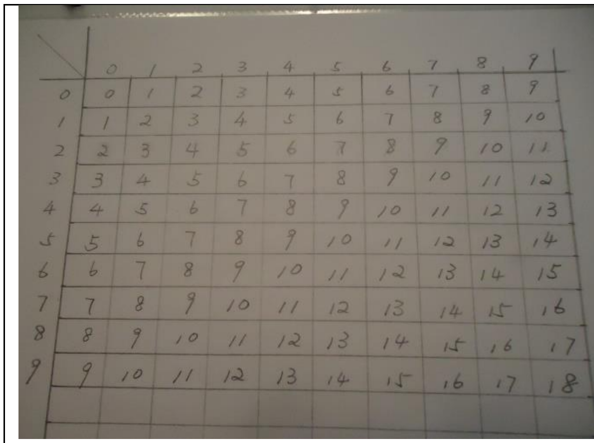
1) the single-digit to single-digit addition table;
2) the single-digit to single-digit multiplication table;
3) the set of rules on the decimal point – how to move the decimal point to facilitate calculation and what the steps imply;
4) the set of rules on signs – to determine whether the result is positive or negative;
5) Warning: in doing division, if the divisor is longer than one digit, trials-and-errors involving backtracking until the correct digit is found in each step along the path in obtaining the final result will be needed. [As a reminder, this is exactly what one is doing in ‘long-division.’]

The reason why abacus works -- as well as why long-hand arithmetic works – is because no matter how complicated the operation is, it is broken down into simple steps, in each of which the user is pitting one single digit of a number against one single digit of another number.

[2] The benefits of learning mathematics are certainly not limited to its applications in science and technology, or to be able to economize while shopping at the supermarket. Most importantly, learning mathematics trains the students in building up a logical and independent mind. In this information age, while disinformation is rampaging, having a logical and independent mind is crucial to guarantee the future of democracy and everything else in a democracy, such as social justice. As obvious as it is, I still want to point out that only when majority of people have a logical and independent mind, can a suitable President be elected. [This note is updated 10/27/2020.]

The simplicity guarantees that the result is correct as long as each step is carried out correctly.

I will not show you the two sets of rules here. To make it more friendly, the rules will be mentioned in examples here and there and will be concluded into two sets of rules in the end. But I will show you the two single-digit to single-digit tables first:



	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 15: the single-digit to single-digit addition table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Figure 16: the single-digit to single-digit multiplication table

Solving Problem #1

Problem #1: Kat works at Walmart in the evening and during weekend. For the past two weeks, she clocked 22 hours. Kat is being paid $10.25/hr. How much will she be paid for these two weeks?

Solution:

1) It is a multiplication problem, because it is asking she is being paid $10.25 per hour, how much she will be paid for 22 hours.
2) The decimal point is in the number 10.25 -- The easiest way to do is to make it a whole number by pushing the decimal point to the very right so the number becomes 1025.
3) When we push the decimal point to the right across each digit, we are actually multiplying the number by 10. Here, for example, we pushed it two digits to the right, we multiplied the original number by 10 two times, which was 100. (10.25 X 100 = 1025)
4) Before the end, we must remember to push the decimal point backward two places so that the result will be correct.
5) Calculating 1025 X 22 longhand with pencil and paper: 22550 (Figure 17)
6) Calculating 1025 X 22 longhand in abacus: 22550 (Figure 18)

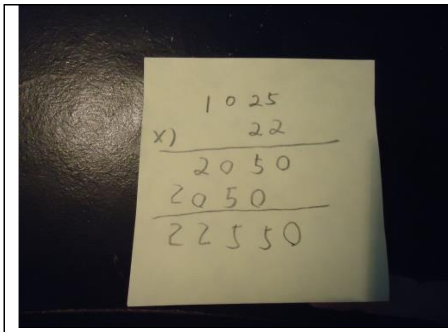


Figure 17: longhand with pencil and paper


Figure 18: longhand by abacus with beads moved to their final positions

7) Pushing the decimal point backward two places: 225.5
8) Checking answer with TI-nSPIRE

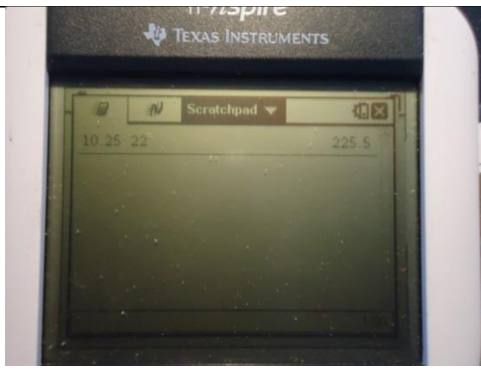


Figure 19: answer is checked correct

Steps and Sub-steps Further Explained

In the following, the steps and sub-steps done in the abacus are further explained:

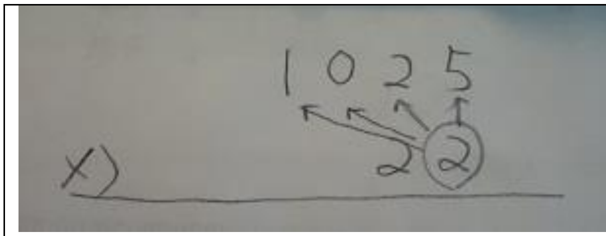


Figure 20: operations of the 1's digit of the multiplier to each digit in the number to be multiplied

Figure 21: 2 X 5 = 10 (add 0 to original digit and a carry of 1 to the digit higher up)

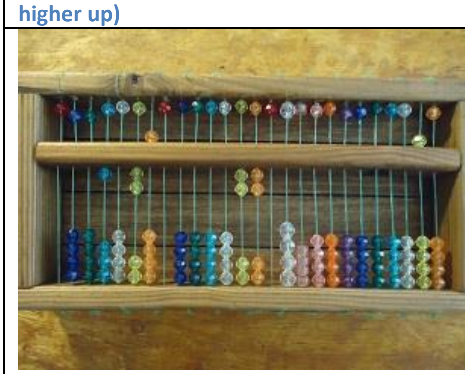


Figure 22: 2 X 2 = 4 (4 plus the 1 originally there in the 10's position to become 5)

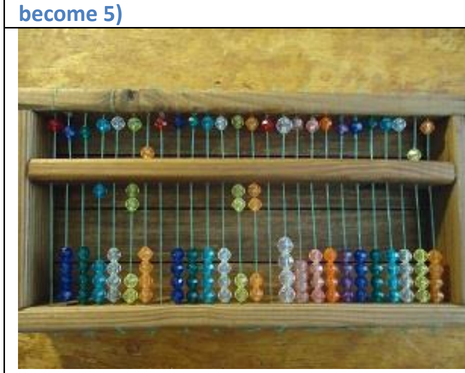


Figure 23: 2 X 0 = 0 (add nothing)

Figure 24: 2 X 1 = 2 (add 2 to the thousand's position to result in 2050)

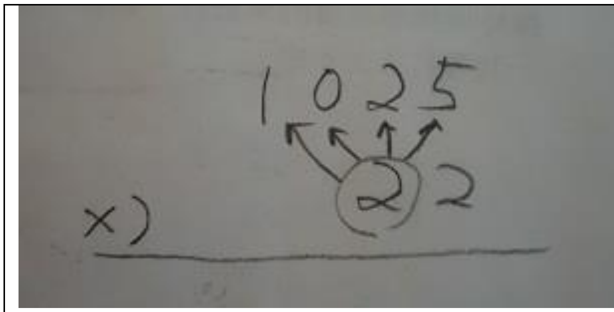


Figure 25: operations of the 10's digit of the multiplier to each digit in the number to be multiplied


Figure 26: 2 X 5 = 10 (add 1 to digit higher up than its own 10’s position)

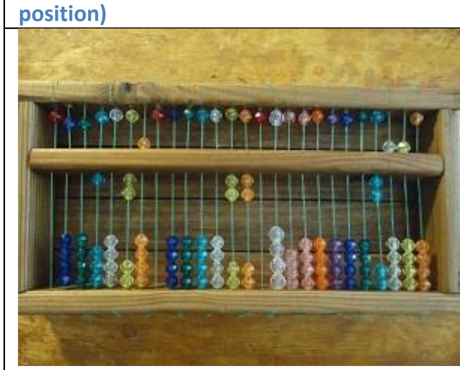


Figure 27: 2 X 2 = 4 (add 4 to the originally 1 there in the 100’s position to become 5)

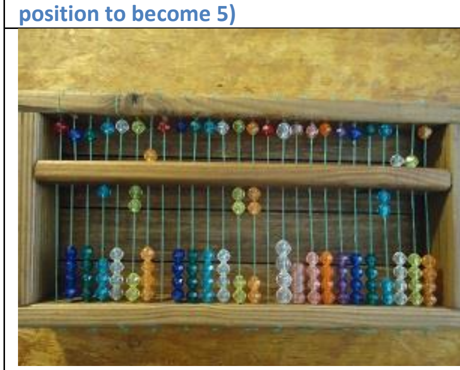


Figure 28: 2 X 0 = 0 (nothing is added)


Figure 29: 2 X 1 = 2 (add 2 to the ten thousand’s position to result in 22550)

Chapter 4: Problem #2: the Monthly Investment

Division might be easier than multiplication.

In our previous lesson, we did a multiplication problem. Today, we will do division. Moreover, if you remember the 'warning' we presented in the previous lesson, it was also on division. But do not let the warning discourage you. With a good choice of the divisor, even if it is longer than one digit, the problem can still be solved with ease and fun!

Problem #2: Kat plans to save $3078 a year. Not counting any interest, how much does she have to save monthly?

Solution:

1) This is a division problem. It is dividing a yearly saving of $3078 into 12 monthly savings without considering any interest that may incur, how much does she have to save monthly? (Figure 30.)

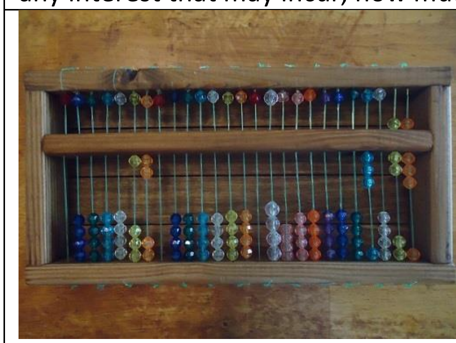


Figure 30: 3078 to be divided by 12

2) It will be of great help to know that 12, 24, 36, 48, 60, 72, 84, and 96 are all exact multiples of 12.

3) We are to divide by more than 10, therefore, we immediately know that the quotient will be at most 3-digit long.

4) Looking at the first two digits of the number to be divided, 30, we make the decision that 2 will be the first digit (hundred's position) of our quotient because 12 X 2 = 24. We take 24 away from 30 to have 6 remained. (Figure 31.)

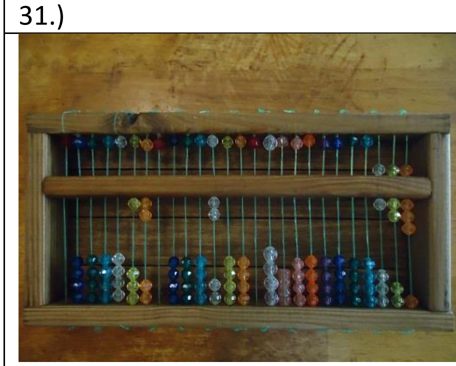


Figure 31: 2 is chosen as the first digit of the quotient

5) Now we look at the first two digits of the remaining 678. We know that 5 will be our choice, because 12 X 5 = 60. We take away 60 from 67 to have 7 remained. (Figure 32.)


Figure 32: 5 is chosen as the second digit of the quotient

6) Now we look at the remaining 78 and determine that 6 will be our choice, because 12 X 6 = 72. We take 72 away from 78 to have 6 remained. (Figure 33.)


Figure 33: 6 is chosen as the third digit of the quotient

7) Answer: the quotient is 256 and the remainder is 6.

8) In other words, Kat has to save $256 each month, with one month in which she has to save $6 additionally. Or she can divide $6 into 12 months, and save ¢50 more each month.

9) We check the answer by using TI-nSPIRE to verify it is correct. (Figure 34.)

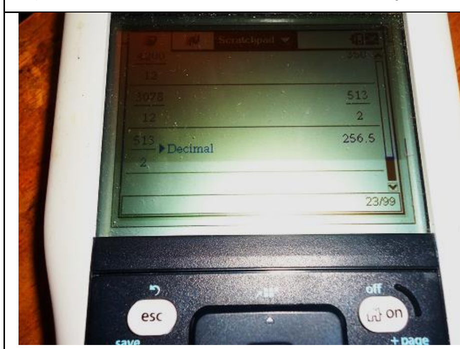


Figure 34: remainder is 6 which is equivalent to .5 of 12

Chapter 5: the Kongfu Fingers Game

We learned from the previous chapters that the particular feature that makes the Chinese abacus easy to work is that in each column there is a special bead -- the one above the bar -- that represents 5. With that special bead, each column's value can be represented with only having 5 beads to cover 0 through 9.

With this particular feature in mind, I want you to design a game that will make it possible for you to use your two hands to signal the values from 0 through 99. Will you be able to do that? If your answer is yes, then how do you do that?

I will give you the answer in the next chapter.

Chapter 6: Answer to the Kongfu Fingers Game

The question was, in the previous chapter, how do we use our two hands to represent the numbers from 0 all the way to 99?

The 'place values' now come into play. If we do not have the concept of place values, then all we can do with our two hands is to represent the numbers from 0 to 10. However, if we 1) let our right hand have the place value of 1, and the left hand have the place value of 10, and 2) let our thumbs to represent the number '5' (the same way as the Chinese abacus works), won't we able to use our two hands to represent the numbers from 0 all the way through 99 now? The following pictures will show why.


Figure 35: 0 X 10 = 0


Figure 36: 0 X 1 = 0


Figure 37: 1 X 10 = 10


Figure 38: 1 X 1 = 1


Figure 39: 2 X 10 = 20

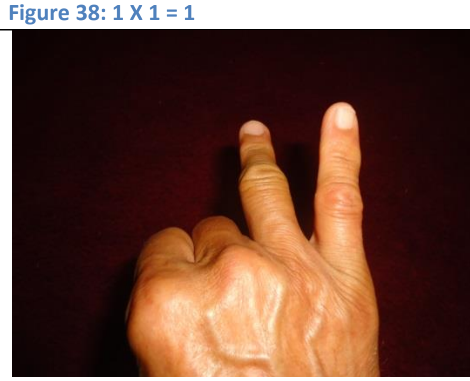


Figure 40: 2 X 1 = 2


Figure 41: 3 X 10 = 30


Figure 42: 3 X 1 = 3

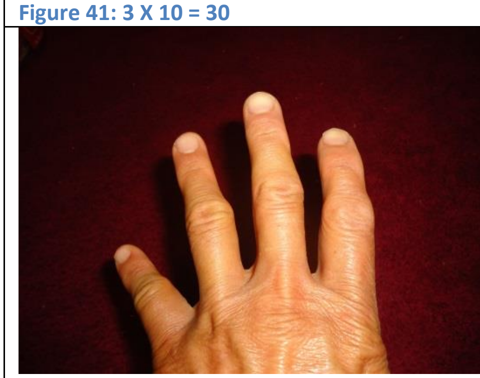


Figure 43: 4 X 10 = 40

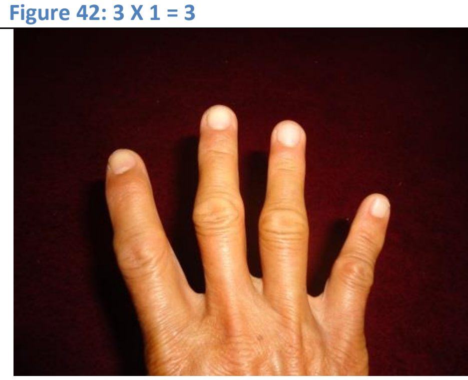


Figure 44: 4 X 1 = 4

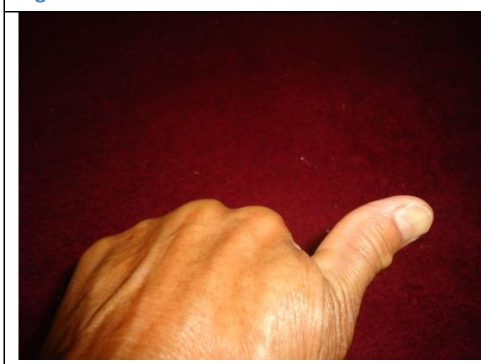


Figure 45: 5 X 10 = 50


Figure 46: 5 X 1 = 5

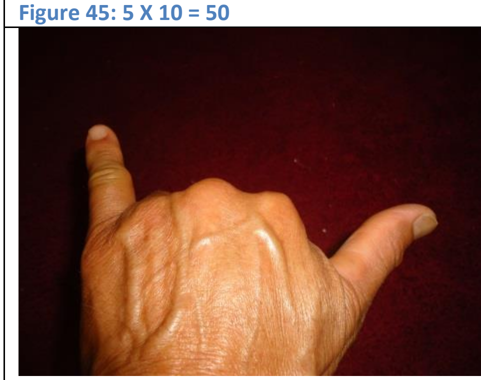


Figure 47: 6 X 10 = 60


Figure 48: 6 X 1 = 6


Figure 49: 7 X 10 = 70


Figure 50: 7 X 1 = 7

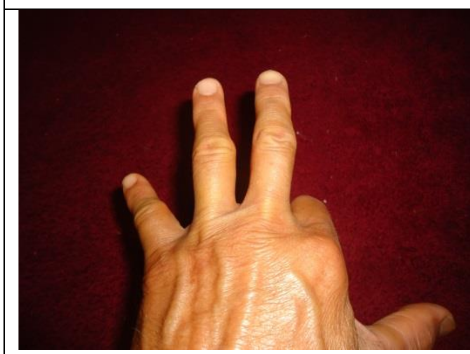


Figure 51: 8 X 10 = 80

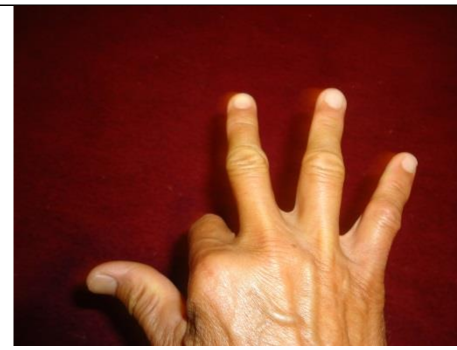


Figure 52: 8 X 1 = 8


Figure 53: 9 X 10 = 90

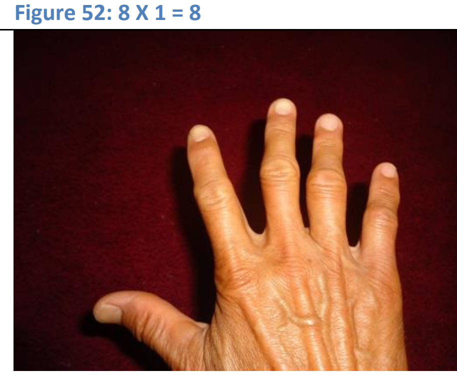


Figure 54: 9 X 1 = 9

With the place values in mind, we will be able to represent the numbers from 0 all the way through 99, as the above pictures have just shown us. Name any number and combine your two hands to represent it.

Now, another challenge: We have four students come to the board and let their two hands represent different place values. What will be the smallest and the largest numbers they can represent with their hands? Also, will they be able to represent any number between these two extremes? (**The answers are 0 and 99,999,999. Try to figure out why on your own. And, of course, they will.**) It will be easier if they stand shoulder to shoulder with their backs facing the class and stretch their hands and fingers upward.

If the class like even more challenges, let another student volunteer to go to the board to represent the positive or negative sign (with any gestures). Now, what will be the largest number and the smallest?

Conclusions

I have mentioned that the purpose of my encouraging the students to use the abacus is not for it to take the place of a calculator. Nor is it my focus for it to compete with a calculator. On the contrary, my purpose is for them (an abacus and a calculator) to complement each other, for the reasons as follows:

Calculators have become so easy to use and so powerful that students can obtain the results they need without having to understand the basic mathematics rules and operations. All they need to do is to punch the buttons. However, when they are facing more advanced mathematics topics, they are immediately frustrated because they did not build up the fundamental knowledge they now need.

An abacus does just that: it can train the students through hands-on, step-by-step, operations.

I also mentioned that along the way, we would build the set of rules that we need to take care of how to determine the sign (positive or negative) of the result and also how to take care of the moving of the decimal point.

The rules to determine the positive or negative nature of the result are as follows:

A positive number plus another positive number.	The result is positive.
A positive number plus a negative number. (Or, a negative number plus a positive number.) (**Addition is a commutative operation.**)	Determine by looking at the absolute value of the numbers. If the absolute value of the positive number is larger, then the result is positive. If the absolute value of the negative number is larger, then the result is negative.
A negative number plus another negative number.	The result is negative.
A positive number minus another positive number.	If the former is larger, the result is positive. If the former is smaller, the result is negative.
A positive number minus a negative number.	The result is positive. It is the same as the positive number plus the absolute value of the negative number.
A negative number minus a positive number.	The result is negative.
A negative number minus another negative number.	It is the same as adding the absolute value of the second number. If this absolute value is larger than the absolute value of the first number, then the result is positive. If this absolute value is smaller than the absolute value of the first number, then the result is negative.
A positive number times another positive number.	The result is positive.
A positive number times a negative number. (Or, a negative number times a positive number.) (**Multiplication is a commutative operation.**)	The result is negative.
A negative number times another negative number.	The result is positive.
A positive number divided by another positive number.	The result is positive.
A positive number divided by a negative number.	The result is negative.
A negative number divided by a positive number.	The result is negative.
A negative number divided by another negative number.	The result is positive.

The rules for moving the decimal point are as follows:

For addition and subtraction	Line up the decimal point in the numbers to be operated on. Look at the arrangement of the colors of the beads so you can designate the place value for each of them to your best advantage.
For multiplication	If, during a calculation, you have moved the decimal point to the right for easier operations, always move it backward however many places you have moved it to the right accumulatively.
For division	Treat the dividend the same way as a number in multiplication. On the other hand, if you have moved the decimal point in the divisor to the right in an operation, then you have divided more that many times by 10. So, in the end, you need to move the decimal point to the right by that many places to return to the result’s correct value[3] .

These two sets of rules apply to any arithmetic calculation. They are not there only when one uses an abacus. In other words, when you operate an abacus, you are training yourself in these rules also.

As mentioned in the beginning chapter, there is a way for the Texas abacus to designate the positive or negative sign of a number. Utilize it by all means. By looking at the numbers to be operated on, including their signs, the user can determine the positive or negative nature of the end result already even before any other calculation begins.

With these and what we covered in previous chapters, students will be able to confidently operate an abacus. And through the process, slowly but surely, they will understand and become familiar with the basic mathematics rules and operations, which will, in their turn, pave the way for the students to go further and further in mathematics with clarity, precision, and confidence.

[3] **The behavior of the divisor is the opposite that of the dividend.**